

## Special limits and nonrelativistic solutions

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### Abstract

We study special vanishing horizon limit of ‘boosted’ black D3-branes having a compact light-cone direction. The type IIB solution obtained by taking such a zero temperature limit is found to describe a nonrelativistic system with dynamical exponent 3. We discuss about such limits in M2-branes case also.

# 1 Introduction

There are mainly two type of non-relativistic string backgrounds, with broken Lorentzian symmetry, which have been a subject of favorable attention currently [1]-[20]. The ones which exhibit Schrödinger symmetries [1, 2] are written as

$$ds_{Sch}^2 = \left( -\frac{\beta^2}{z^{2a}}(dx^+)^2 - \frac{dx^+ dx^-}{z^2} + \frac{dx_i^2}{z^2} \right) + \frac{dz^2}{z^2} \quad (1)$$

and the others with Lifshitz-like symmetries [3, 4] are

$$ds_{Lif}^2 = \left( -\frac{\beta^2}{z^{2a}}(dt)^2 + \frac{dx_i^2}{z^2} \right) + \frac{dz^2}{z^2}. \quad (2)$$

In both,  $x^i$  ( $i = 1, \dots, d$ ) are spatial flat coordinates,  $z$  is the holographic scale and  $a$  is called the dynamical exponent. These geometries have been claimed to be describing interesting scaling phenomena near quantum critical points in the dual CFTs [1, 2]. The Schrödinger AdS spacetimes can be embedded in string theory as was shown in [5, 6]. These spaces can even be obtained in the massive type IIA string theory [15]. More recently, the Lifshitz-like spaces have been embedded in string theory as demonstrated by [18] and generalised by [19]. This implies that a wide class of non-relativistic solutions can be found in string theory and the hope is that these could potentially describe interesting scaling phenomena in dual field theories on the boundary. For study of finite temperature properties like phase transitions, entropy etc. one includes black holes in the AdS backgrounds.

In this work we shall discuss a new type of Galilean AdS geometry which has one of the lightcone direction, namely  $x^+$ , being null while the other,  $x^-$ , being compact

$$ds^2 = \left( -\frac{dx^+ dx^-}{z^2} + \frac{\beta^2 z^2}{4}(dx^-)^2 + \frac{dx_1^2 + dx_2^2}{z^2} \right) + \frac{dz^2}{z^2} \quad (3)$$

It is a unique non-relativistic solution directly obtainable by taking ‘zero’ temperature (condensation) limits of a boosted black D3-brane AdS geometry. Indeed we find that the corresponding thermodynamic quantities in the dual theory undergo a kind of condensation where

$$T \sim 0, \quad \mu_N \sim 0, \quad s \sim 0, \quad \rho \sim \frac{T^4}{\mu_N^3} = \text{fixed}. \quad (4)$$

Our study shows that such thermodynamic limits do exist which directly lead us to such non-relativistic Lifshitz systems at zero temperature. The zero temperature non-relativistic theory dual to the Galilean geometry (3) however exhibits a scaling symmetry with dynamical exponent  $a = 3$ .

The paper is organised as follows. In section-II we describe the boosted black D3-brane AdS solution. We then go over to discuss a special  $r_0 \rightarrow 0$ ,  $\lambda \rightarrow \infty$  limit

in which the black hole horizon is allowed to shrink while the boost is simultaneously taken to be very large. The geometry thus obtained describes a ‘zero’ temperature nonrelativistic dynamics in a 2-dimensional (planar) quantum mechanical system. In section-III we discuss a type IIA Lifshitz solution obtained under T-duality. The section-IV consists of taking similar limits of the boosted black M2-branes. The dual theory represents a nonrelativistic phenomenon with fractional scaling power in one space dimension. The conclusions are given in section-V.

## 2 Vanishing horizon limits

### 2.1 DLCQ and $AdS_5$ space

We are interested in studying the DLCQ of the  $AdS_5$  space as described in [6]. We start with the boosted version of black D3-branes [6] where the near horizon geometry is

$$\begin{aligned}
 ds_{D3}^2 &= r^2 \left( -\frac{1+f}{2} dx^+ dx^- + \frac{1-f}{4} \left[ \frac{(dx^+)^2}{\lambda^2} + \lambda^2 (dx^-)^2 \right] + dx_1^2 + dx_2^2 \right) \\
 &\quad + \frac{dr^2}{f r^2} + d\Omega_5^2, \\
 F_5 &= 4(1 + *) Vol(S^5)
 \end{aligned} \tag{5}$$

where  $d\Omega_5^2$  represents the line element of a unit five-sphere while  $Vol(S^5)$  is its volume element. The overall  $AdS_5$  radius has been set to unity. Here  $f(r) = 1 - r_0^4/r^4$  with  $r = r_0$  being the horizon location and the boundary is at  $r \rightarrow \infty$ . The boundary conformal field theory has finite temperature. These black 3-branes have large (finite) momentum along  $x^-$ , which is compact,  $x^- \sim x^- + 2\pi r^-$ . If we try to set  $r_0 = 0$  it would make  $x^-$  circle null for which we should be careful. It usually is not a problem when  $x^-$  is noncompact, because then setting  $r_0 = 0$  simply is the extremal limit which gives ordinary AdS spacetime whose dual is a super-Yang-Mills theory at large 't Hooft coupling,  $g_{YM}^2 N'$ . ( $N'$  is the order of the gauge group and also the number of D3-branes which give rise to the AdS geometry.) Note that the size of  $x^-$  circle anyway vanishes as we go near the boundary at  $r \rightarrow \infty$ , but nevertheless it stays finite in the region inside the bulk. Physically, the boost (scale) parameter  $\lambda$  controls the size of  $x^-$  circle.

### 2.2 The simultaneous $r_0 \rightarrow 0$ , $\lambda \rightarrow \infty$ limit

As we learnt, taking  $r_0 = 0$  is not possible in (5) when  $x^-$  is compact. But, since  $r_0^4 \lambda^2$  effectively controls the size of the circle we would instead consider a simultaneous limit in which the size of horizon is allowed to shrink while boost is taken to infinity

such that

$$r_0 \rightarrow 0, \quad \lambda \rightarrow \infty, \quad r_0^4 \lambda^2 = \beta^2 = \text{fixed}. \quad (6)$$

In which case we can have

$$\begin{aligned} (1+f) &\rightarrow 2 + O(r_0^4), & \frac{1-f}{\lambda^2} &\rightarrow O\left(\frac{r_0^4}{\lambda^2}\right) \\ (1-f)\lambda^2 &\rightarrow \frac{\beta^2}{r^4} \end{aligned} \quad (7)$$

and the solution (5) reduces to

$$\begin{aligned} ds_{D3}^2 &\simeq r^2 \left( -dx^+ dx^- + \frac{\beta^2}{4r^4} (dx^-)^2 + dx_1^2 + dx_2^2 \right) + \frac{dr^2}{r^2} + d\Omega_5^2 \\ F_5 &= 4(1+*)Vol(S^5) \end{aligned} \quad (8)$$

which is a complete solution of type IIB string theory. The coordinate  $r$  can be identified with  $1/z$  in (3). Actually spacetime (8) is an  $AdS_5$  geometry but it is Galilean one and with  $x^+$  being null. However it should not be worried since  $x^-$  is a circle and we can always rewrite Eq.(8) in a diagonal basis as

$$\begin{aligned} ds_{D3}^2 &= r^2 \left( -\frac{r^4}{\beta^2} (dx^+)^2 + \frac{\beta^2}{4r^4} (dx^- - \frac{2r^4}{\beta^2} dx^+)^2 + dx_1^2 + dx_2^2 \right) + \frac{dr^2}{r^2} + d\Omega_5^2 \\ &= \left( -\frac{r^6}{\beta^2} (dx^+)^2 + r^2 (dx_1^2 + dx_2^2) + \frac{dr^2}{r^2} \right) + \frac{\beta^2}{4r^2} (dx^- - \frac{2r^4}{\beta^2} dx^+)^2 + d\Omega_5^2 \end{aligned} \quad (9)$$

From above eq.(9) it is obvious that the Galilean geometry (8) indeed represents a well defined system of Kaluza-Klein particles if we ever dimensionally reduce over  $x^-$  and go to lower dimensions. That is we are simply dealing with bosonic KK excitations in one less dimensions. However, unlike (5) the new geometry (8) is inherently nonrelativistic. The line element  $\left( -\frac{r^6}{\beta^2} (dx^+)^2 + r^2 (dx_1^2 + dx_2^2) + \frac{dr^2}{r^2} \right)$  in eq.(9) is precisely the Lifshitz four-universe. Hence we find that by performing the special vanishing horizon limit (6) on the ‘boosted’ black D3-branes allows us to exclusively zoom onto a KK system in a Lifshitz universe. There is a non-relativistic scale (dilatation) invariance

$$r \rightarrow (1/\xi)r, \quad x^- \rightarrow \xi^{2-a}x^-, \quad x^+ \rightarrow \xi^a x^+, \quad x_{1,2} \rightarrow \xi x_{1,2} \quad (10)$$

with dynamical exponent  $a = 3$ . There are also invariances under constant shifts (translations) like

$$x^+ \rightarrow x^+ + b^+, \quad x^- \rightarrow x^- + b^-, \quad x^i \rightarrow x^i + b^i \quad (11)$$

as well as rotations in the  $x^1 - x^2$  plane

$$x^i \rightarrow \omega_j^i x^j. \quad (12)$$

However, (8) does not have any explicit invariance under the Galilean boosts

$$x^+ \rightarrow x^+, \quad x^- \rightarrow x^- - \vec{v} \cdot \vec{x} + \frac{v^2}{2} x^+, \quad \vec{x} \rightarrow \vec{x} - \vec{v} x^+. \quad (13)$$

Same is the case with the special conformal transformations.

Thus our solution (8) represents a (Lifshitz) geometry with a broken Lorentzian symmetry in which the time scales with a dynamical exponent  $a = 3$ . Note, however, no extra matter fields are present in the background (8) except the self-dual 5-form flux. This is unlike many other Schrödinger geometries, e.g. in [5, 6], obtained via T-s-T mechanism, where  $B_{\mu\nu}$  field contributes as ‘dust’ to the energy-momentum tensor. But we do have Kaluza-Klein excitations present in (8). We have checked that the background (8) preserves at least 8 Poincaré supersymmetries.

### 2.3 Limits of thermodynamic quantities

Having obtained the nonrelativistic geometry in (8) we shall now study the effect of the  $r_0 \rightarrow 0$ ,  $\lambda \rightarrow \infty$  limits on the thermodynamic quantities. As we know that the hot boundary CFT involves a DLCQ description [6]. The isometry along  $x^-$  implies that there is a conserved momentum (charge)  $P_-$  in the theory which is quantized in units of  $\frac{1}{r^-}$ . The number (charge) density depends upon the choice of two parameters, namely  $\lambda$  and  $r_0$ . We want to know what happens to the number density, energy density ( $-P_+$ ) and other thermodynamical quantities; like temperature ( $T$ ), entropy ( $S$ ) and chemical potential ( $\mu_N$ ) relevant for the black-hole solution (5) as we consider special limits (6). We can find these estimates simply by using the thermodynamic expressions given in [6]

$$\begin{aligned} \rho &= \frac{N}{v_2} = \frac{r^- (-P_-)}{v_2} = \frac{L^3 (r^-)^2 \lambda^2 r_0^4}{G_N^5 8} \sim \frac{L^3 (r^-)^2 \beta^2}{G_N^5 8} \\ \mathcal{E} &= \frac{H}{v_2} = \frac{(-P_+)}{v_2} = \frac{L^3 r^- r_0^4}{G_N^5 16} \sim O(r_0^4) \sim 0 \\ s &= \frac{S}{v_2} = \frac{L^3}{4G_N^5} (2\pi r^-) \frac{\lambda r_0^3}{2} \sim O(r_0) \sim 0 \\ T &= \frac{r_0}{\pi \lambda} \sim O(r_0^3) \sim 0, \quad \mu_N = \frac{1}{r^- \lambda^2} \sim O(r_0^4) \sim 0 \end{aligned} \quad (14)$$

where  $L(=1)$  is the  $AdS_5$  radius,  $G_N^5$  is 5-dimensional Newton’s constant and  $v_2$  is the volume of  $x_1-x_2$  plane. We see that under our special limits (6), the temperature of boundary  $(2+1)$  dimensional theory effectively vanishes so also the entropy and the chemical potential. Curiously though, we find that the number density remains fixed while energy density altogether vanishes. Although these quantities are vanishing, worth noticing is their unique scaling behaviour as powers of vanishing horizon radius  $r_0$

$$T \sim r_0^3, \quad \mathcal{E} \sim r_0^4, \quad \mu_N \sim r_0^4, \quad s \sim r_0 \quad (15)$$

The temperature vanishes as  $r_0^3$  which is an indication of the fact that system becomes nonrelativistic with dynamical exponent 3 as it undergoes condensation.<sup>1</sup> Thus it would be appropriate to call the ‘vanishing’ horizon limits (6) as ‘zero’ temperature (or condensation) limits

$$T \rightarrow 0, \quad \mu_N \rightarrow 0, \quad \rho \sim \frac{T^4}{\mu_N^3} = \text{fixed.} \quad (16)$$

in the DLCQ theory. Moreover the scaling behaviour in (15) summarises how this condensation would be achievable such that at the end point we have a zero temperature nonrelativistic DLCQ theory with dynamical exponent  $a = 3$ . In this sense this zero temperature limit of the DLCQ is very special. To recall the thermal  $D = 4$  super Yang-Mills theory, dual to nonextremal D3-branes, has a zero temperature (extremal) limit  $r_0 \rightarrow 0$  under which  $T \rightarrow r_0$ ,  $s \rightarrow r_0^3$ . The end point of this condensation leads us to ordinary  $\mathcal{N} = 4$  super Yang-Mills theory which is a relativistic theory.

If we wish, from eqs.(14) we can also write down the standard thermodynamical expressions for free energy density and the entropy density

$$F \sim -\mathcal{E} \sim -\frac{T^4}{\mu_N^2} \sim 0, \quad s \sim \frac{T^3}{\mu_N^2} \sim 0 \quad (17)$$

which are vanishing under our zero temperature limit. So this describes a condensate in DLCQ and there is only one relevant parameter and that is overall number density ( $\rho$ ) in a given condensate which depends on  $\beta$ .<sup>2</sup> We have the following comments to offer.

- The thermodynamic quantities in (14) usually satisfy the first law of thermodynamics

$$\delta H(T, \mu_N) = T\delta S - \mu_N \delta N \quad (18)$$

Since both  $(T, \mu_N) \rightarrow 0$  we see that energy fluctuations  $\delta H$  are very much suppressed. This provides stability to the ground state under thermal fluctuations.

- Note, our classical geometry (8) cannot be trusted near the AdS boundary as the physical size of  $x^-$  circle

$$\frac{R_{phys}^-}{l_s} = \frac{L}{l_s} \frac{r_0}{r} \frac{r^- \lambda r_0}{2} \sim \beta \frac{r^-}{r} \quad (19)$$

vanishes near  $r \rightarrow \infty$ . It will then be appropriate to go over to type IIA string picture where the T-dualised circle could have a finite radius. We shall discuss more about it next.

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<sup>1</sup>In the DLCQ set-up [6], the lightcone Hamiltonian in a given momentum sector ( $P_-$ ) reads as  $H = -P_+ \sim \frac{\vec{p}^2}{(-4P_-)} + \dots$ . Here conserved momentum ( $-P_-$ ) plays the role of Galilean mass  $M$ .

<sup>2</sup> An interesting study on condensation in Galilean ideal gases can be found in [21].

- We have a well controlled zero temperature limits both within the bulk and in the boundary theory. Thus we can safely conclude that the zero temperature geometry (8) is genuine and its dual boundary theory will exhibit a Lifshitz symmetry with dynamical exponent  $a = 3$  at least in the IR region. In the UV region we will have to look for an appropriate M-theory picture as we discuss it next.

### 3 The type IIA solution

We noticed that the size of the compact null direction in type IIB solution (8) becomes sub-stringy as we approach the boundary region. So it would be appropriate to make T-duality transformation along the  $x^-$  circle in (8) where its radius gets inverted. We write down corresponding type IIA background as

$$\begin{aligned}
ds_{IIa}^2 &= \left( -\frac{r^6}{\beta^2} (dx^+)^2 + r^2 \left( \frac{4}{\beta^2} (d\tilde{x}^-)^2 + dx_1^2 + dx_2^2 \right) + \frac{dr^2}{r^2} \right) + d\Omega_5^2 \\
&= \left( -\frac{1}{\beta^2 z^6} (dx^+)^2 + \frac{dx_1^2 + dx_2^2 + dz^2}{z^2} \right) + \frac{4}{\beta^2 z^2} (d\tilde{x}^-)^2 + d\Omega_5^2, \\
e^{2\phi_a} &= g_s^2 \left( \frac{4}{\beta^2 z^2} \right), \quad A_{+12} \sim -\frac{1}{z^4}, \quad B_{-+}^{NS} \sim \frac{2}{\beta^2 z^4}
\end{aligned} \tag{20}$$

The circle  $\tilde{x}^-$  is the T-dual circle in type IIA. The above solution has a geometry in which a 4-dimensional Lifshitz spacetime has a circle  $\tilde{x}^-$  foliated over it. However the dilaton and the NS  $B$ -field are also present. There is a new Lifshitz type scaling invariance

$$z \rightarrow \xi z, \quad x^+ \rightarrow \xi^2 x^+, \quad x_{1,2} \rightarrow \xi x_{1,2} \tag{21}$$

provided that  $\beta \rightarrow \xi^{-1} \beta$ . Note that  $\tilde{x}^-$  does not scale, this scaling is broken in type IIA solution. This is the effect of T-duality along  $\tilde{x}^-$ . But since  $\beta$  is a free parameter effectively controlling the size of transverse  $\tilde{x}^-$  circle, from type IIA point of view the Lifshitz scaling (21) would take us from one type IIA picture to an equivalent type IIA picture.

We see from type IIA solution (20) that the string coupling runs and it becomes large in UV. As we go close to the boundary the type IIA description will break down and one would need to go over to an appropriate M-theory picture. The M-theory geometry obtained by uplifting the solution (20) to eleven dimensions is

$$\begin{aligned}
ds_M^2 &= \left( \frac{\beta^2 z^2}{4g_s^2} \right)^{\frac{1}{3}} \left( -\frac{(dx^+)^2}{\beta^2 z^6} + \frac{dx_1^2 + dx_2^2 + dx_{11}^2 + (d\tilde{x}^-)^2 + dz^2}{z^2} + d\Omega_5^2 \right) \\
&\simeq \left( \frac{\beta^2}{4z^4} \right)^{1/3} \left( \left[ -\frac{1}{\beta^2 z^4} (dx^+)^2 + dx_1^2 + dx_2^2 \right] + dx_{11}^2 + (d\tilde{x}^-)^2 + (dz^2 + z^2 d\Omega_5^2) \right) \\
C_{+12} &\sim \frac{1}{z^4}, \quad C_{+-11} \sim \frac{1}{\beta^2 z^4}
\end{aligned} \tag{22}$$

where circular coordinate  $x_{11}$  has been appropriately scaled. This describes a collection of  $N'$  M2-branes stretched along  $x^1 - x^2$  plane and living over transverse 8-dimensional space  $S^1 \times \tilde{S}^1 \times R^6$ . We observe that the solution (22) still has a singularity near  $z = 0$  as  $(F_{(4)})^2$  would blow up there and it would be inevitable to include quantum corrections.

## 4 Limits for black M2-branes

Similar to the zero-temperature limits of the DLCQ in  $AdS_5$  space, we can also study the similar limits for DLCQ of M2-brane theory. We start with the boosted version of black M2-branes (with a compact lightcone circle) where the near horizon geometry can be written as

$$ds_{M2}^2 = r^4 \left( -\frac{1+f}{2} dx^+ dx^- + \frac{1-f}{4} \left[ \frac{(dx^+)^2}{\lambda^2} + \lambda^2 (dx^-)^2 \right] + dy^2 \right) + \frac{dr^2}{f r^2} + d\Omega_7^2, \\ F_4 = 6 Vol(AdS_4) \quad (23)$$

where  $d\Omega_7^2$  is the line element of a unit seven-sphere and  $\lambda$  is the boost parameter. Here  $f(r) = 1 - \frac{r_0^6}{r^6}$  and  $r = r_0$  is the horizon location for the black M2-branes. Taking the vanishing horizon limits analogous to the D3-brane case

$$r_0 \rightarrow 0, \quad \lambda \rightarrow \infty, \quad r_0^6 \lambda^2 = \beta^2 = \text{fixed}, \quad (24)$$

the geometry (23) becomes

$$ds_{M2}^2 \simeq r^4 \left( -dx^+ dx^- + \frac{\beta^2}{4r^6} (dx^-)^2 + dy^2 \right) + \frac{dr^2}{r^2} + d\Omega_7^2 \\ = \tilde{r}^2 \left( -dx^+ dx^- + \frac{\beta^2}{4\tilde{r}^3} (dx^-)^2 + dy^2 \right) + \frac{d\tilde{r}^2}{4\tilde{r}^2} + d\Omega_7^2, \quad (25)$$

while  $F_4$  flux remains unchanged. The scale invariance could be read as

$$\tilde{r} \rightarrow (1/\xi)\tilde{r}, \quad x^- \rightarrow (1/\sqrt{\xi})x^-, \quad x^+ \rightarrow \xi^{5/2}x^+, \quad y \rightarrow \xi y. \quad (26)$$

There is also shift invariances along  $x^\pm$  and  $y$  but no Galilean boost invariances as such. The geometry (25) would then describe a zero temperature quantum phenomenon along a wire in the Galilean theory. It is interesting to notice that in this M-theory example the time scales with a ‘fractional’ dynamical exponent, which is  $5/2$ .

Once again in the UV region  $\tilde{r} \gg \beta^2/4$  the size of the spatial circle becomes very small in 11-dimensional Planck length units and we cannot trust our classical solution there. It would require to study quantum corrections to the geometry or to add appropriate boundary corrections so as to make it UV picture complete.



## 5 Conclusion

We have studied special ‘vanishing’ horizon limits of the ‘boosted’ black D3-branes having a compact lightcone direction. The limits are taken in such a way that the compact direction does not become null so that we can have DLCQ description of the CFT. The resultant zero temperature solution is found to describe a non-relativistic system with Lifshitz symmetry having a dynamical exponent  $a = 3$ . The new Lifshitz geometry however cannot describe the UV regime as the size of lightcone circle becomes sub-stringy as we approach the AdS boundary. On the other hand the string coupling grows very large in the UV region in a corresponding T-dual type IIA picture, which indicates that it will require us to look for a M-theoretic interpretations. It will therefore be interesting to find a resolution to this UV problem. It would also be worthwhile to explore the zero-temperature limits like in our work for other ‘boosted’ black  $Dp$ -brane solutions and study the resulting Lifshitz solutions thus obtained. We have discussed an example of the black M2-branes here. The Galilean geometry obtained shows that the scaling symmetry has fractional dynamical exponent.

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